

# **Chapter 6**

## **Force and Motion II**

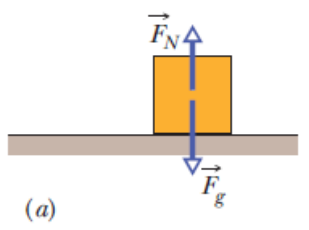
## 6.2 Friction

Frictional forces are common in our everyday lives.

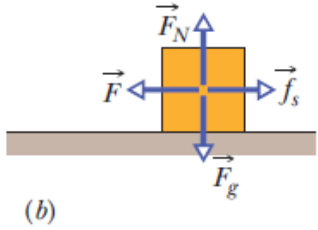
### Examples:

1. If you send a book sliding down a horizontal surface, the book will finally slow down and stop.
2. If you push a heavy crate and the crate does not move, then the applied force must be counteracted by frictional forces.

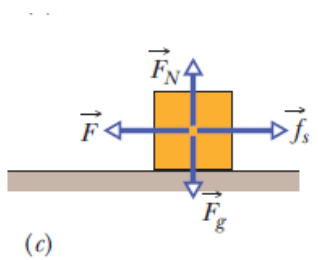
# 6.2 Frictional Force: motion of a crate with applied forces



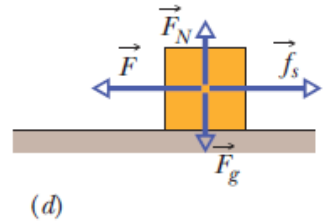
There is no attempt at sliding. Thus, no friction and no motion.  
**NO FRICTION**



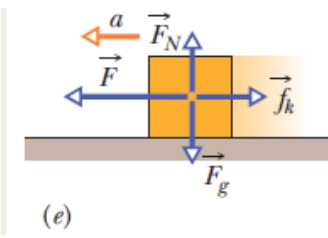
Force  $F$  attempts sliding but is balanced by the frictional force. No motion.  
**STATIC FRICTION**



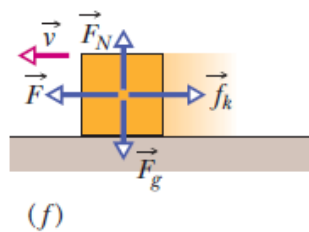
Force  $F$  is now stronger but is still balanced by the frictional force. No motion.  
**LARGER STATIC FRICTION**



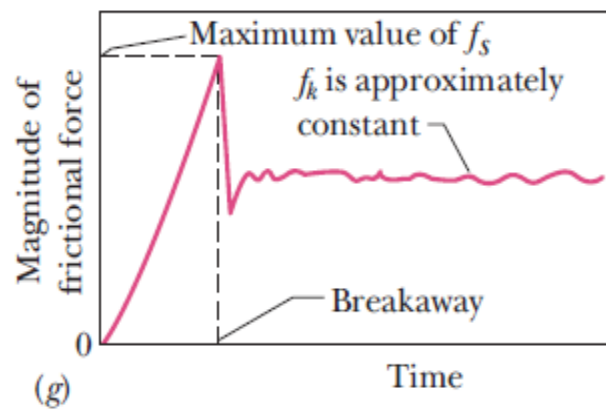
Force  $F$  is now even stronger but is still balanced by the frictional force. No motion.  
**EVEN LARGER STATIC FRICTION**



Finally, the applied force has overwhelmed the static frictional force. Block slides and accelerates.  
**WEAK KINETIC FRICTION**



To maintain the speed, weaken force  $F$  to match the weak frictional force.  
**SAME WEAK KINETIC FRICTION**



Static frictional force can only match growing applied force.

Kinetic frictional force has only one value (no matching).

$f_s$  is the **static frictional force**  
 $f_k$  is the **kinetic frictional force**

## 6.2 Friction

- Static frictional force acts when there is no relative motion between the body and the contact surface
- The magnitude of the static frictional force increases as the applied force to the body is increased
- Finally when there is relative motion between the body and the contact surface, kinetic friction starts to act.
- Usually, the magnitude of the kinetic frictional force, which acts when there is motion, is less than the maximum magnitude of the static frictional force, which acts when there is no motion.
- Often, the sliding motion of one surface over another is “jerky” because the two surfaces alternately stick together and then slip.

### Examples:

- Tires skid on dry pavement
- Fingernails scratch on a chalkboard
- A rusty hinge is forced to open
- A bow is drawn on a violin string

## 6.3 Properties of friction

**Property 1.** If the body does not move, then the static frictional force and the component of  $\mathbf{F}$  that is parallel to the surface balance each other. They are equal in magnitude, and is  $\mathbf{f}_s$  directed opposite that component of  $\mathbf{F}$ .

**Property 2.** The magnitude of has a maximum value  $f_{s,max}$  that is given by

$$f_{s,max} = \mu_s F_N$$


where  $\mu_s$  is the **coefficient of static friction** and  $F_N$  is the **magnitude of the normal force** on the body from the surface. If the magnitude of the component of  $\mathbf{F}$  that is parallel to the surface exceeds  $f_{s,max}$ , then the body begins to slide along the surface.

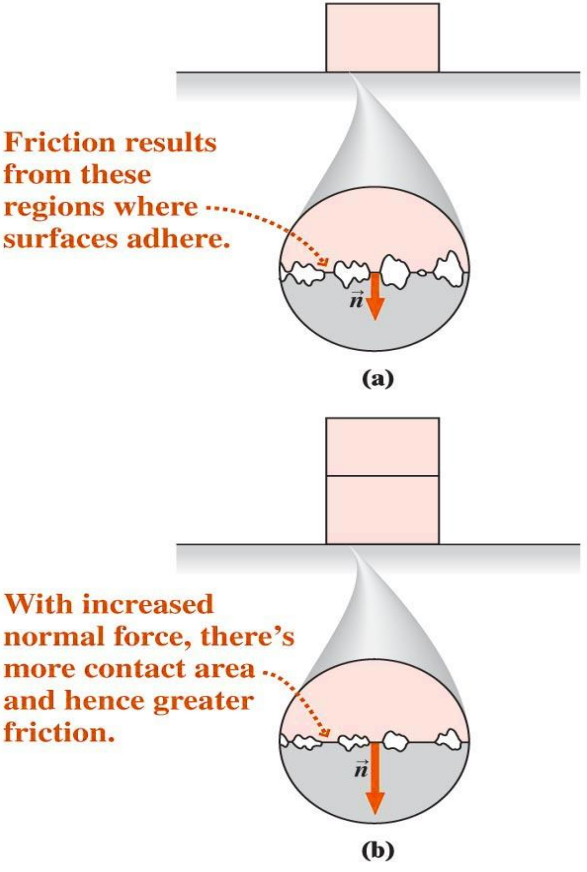
**Property 3.** If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value  $f_k$  given by

$$f_k = \mu_k F_N$$

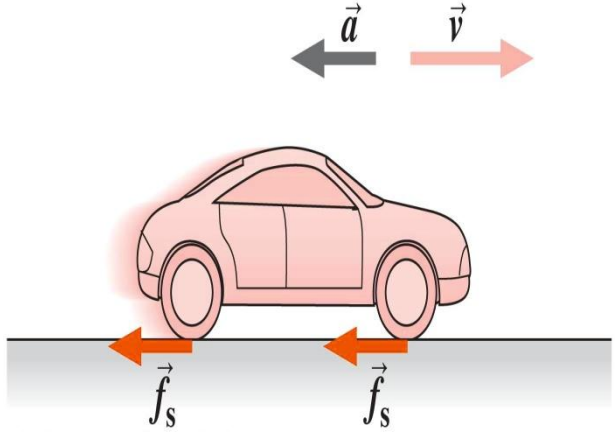
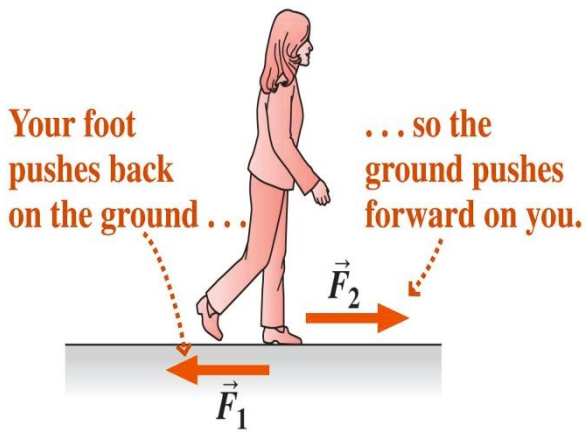
where  $\mu_k$  is the **coefficient of kinetic friction**. Thereafter, during the sliding, a **kinetic frictional force**  $f_k$  opposes the motion.

# 6.3 Properties of friction

- **Friction** is a force that opposes the relative motion of two contacting surfaces.
- **Static friction** occurs when the surfaces aren't in motion; the magnitude is:  $f_s \leq \mu_s F_N$    $F_N$  is the normal force between the surfaces and  $\mu_s$  is the **coefficient of static friction**.
- **Kinetic friction** occurs between surfaces in motion; the magnitude is:  $f_k = \mu_k F_N$ ,  $\mu_k$  is the **coefficient of kinetic friction**.



Friction is important in walking, driving and a host of other applications:

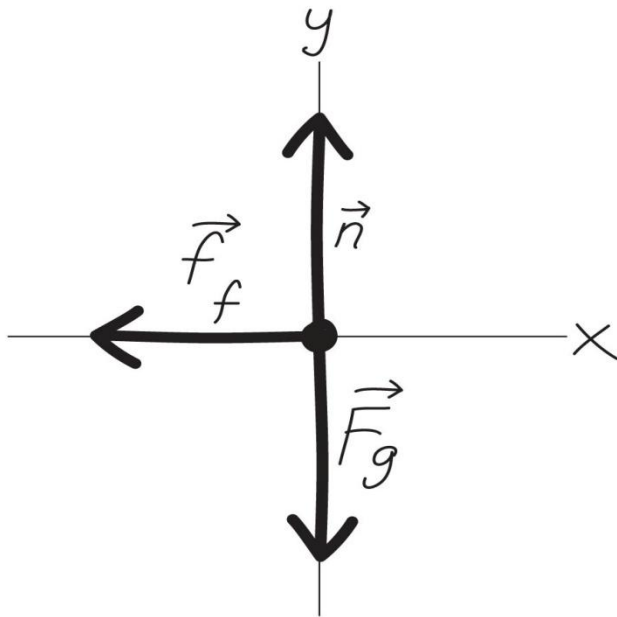


## 6.3 Properties of friction

### Solving Problems with Friction

- Problems with friction are like all other Newton's law problems.
- Identify the forces, draw a free body diagram, write Newton's law.
- You'll need to relate the force components in two perpendicular directions, corresponding to the normal force and the frictional force.

A braking car: What's the acceleration?



### Newton's 2<sup>nd</sup> law:

$$\vec{F}_g + \vec{F}_N + \vec{f}_f = m\vec{a}$$

### *In components:*

$$\sum F_x : -\mu F_N = ma_x$$

$$\sum F_y : -mg + F_N = 0$$

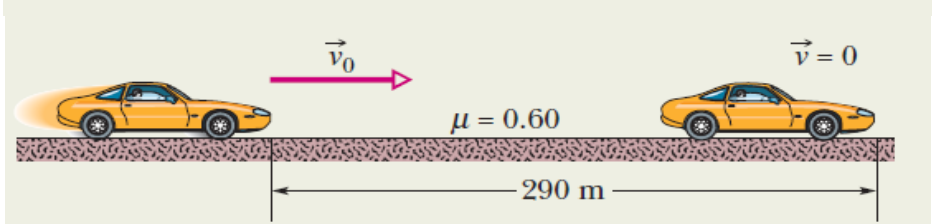
### *Solve for a:*

$$\sum F_y : F_N = mg$$

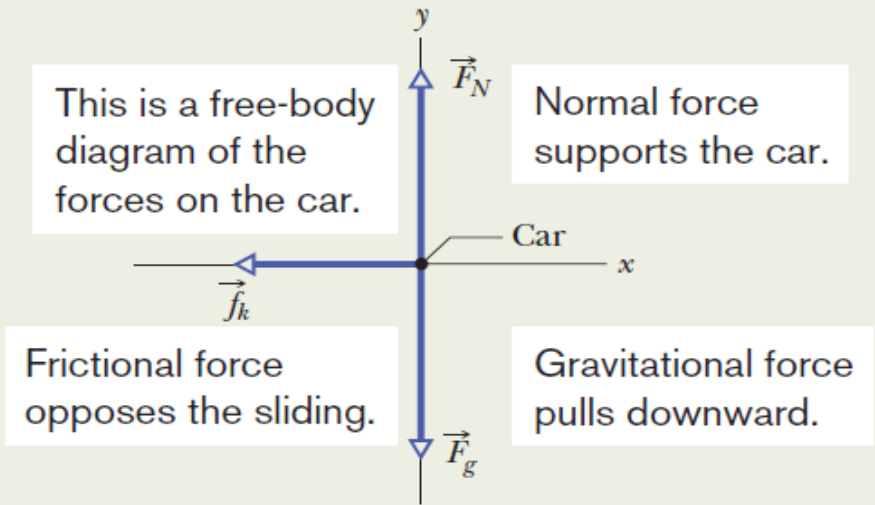
$$\sum F_x : a_x = -\frac{\mu mg}{m} = -\mu g$$

# Example

If a car's wheels are "locked" (kept from rolling) during emergency braking, the car slides along the road. Ripped-off bits of tire and small melted sections of road form the "skid marks" that reveal that cold-welding occurred during the slide. The record for the longest skid marks on a public road was reportedly set in 1960 by a Jaguar on the M1 highway in England (Fig. 6-3a)—the marks were 290 m long! Assuming that  $\mu_k = 0.60$  and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?



(a)



(b)

This is a free-body diagram of the forces on the car.

Normal force supports the car.

Frictional force opposes the sliding.

Gravitational force pulls downward.

Assume that the constant acceleration  $a$  was due only to a kinetic frictional force on the car from the road, directed opposite the direction of the car's motion. This results in:

$$-f_k = ma,$$

where  $m$  is the car's mass. The minus sign indicates the direction of the kinetic frictional force.

**Calculations:** The frictional force has the magnitude

$$f_k = \mu_k F_N,$$

where  $F_N$  is the magnitude of the normal force on the car from the road. Because the car is not accelerating vertically,

$$F_N = mg$$

Thus, 
$$f_k = \mu_k F_N = \mu_k mg$$

$$a = -f_k/m = -\mu_k mg/m = -\mu_k g,$$

where the minus sign indicates that the acceleration is in the negative direction. Use

$$v^2 = v_o^2 + 2a(x - x_o)$$

where  $(x-x_o) = 290$  m, and the final speed is 0. Solving for  $v_o$ ,

$$v_o = \sqrt{2\mu_k g(x - x_o)} = 58 \text{ m/s}$$

We assumed that  $v = 0$  at the far end of the skid marks. Actually, the marks ended only because the Jaguar left the road after 290 m. So  $v_o$  was at least 210 km/h.



## Sample Problem, friction applied at an angle

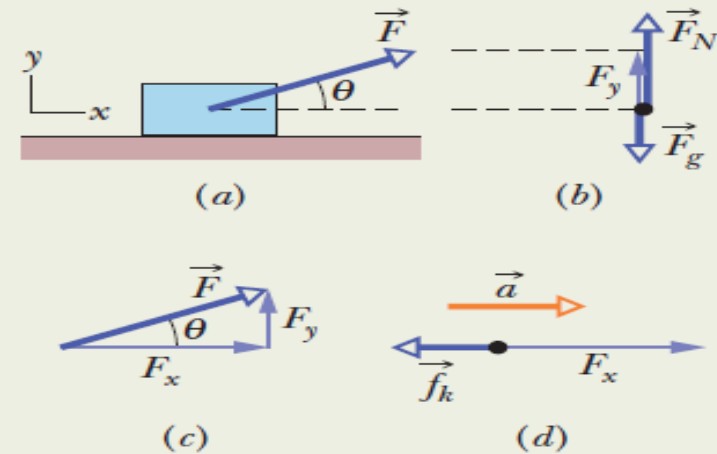
In Fig. 6-4a, a block of mass  $m = 3.0$  kg slides along a floor while a force  $\vec{F}$  of magnitude 12.0 N is applied to it at an upward angle  $\theta$ . The coefficient of kinetic friction between the block and the floor is  $\mu_k = 0.40$ . We can vary  $\theta$  from 0 to  $90^\circ$  (the block remains on the floor). What  $\theta$  gives the maximum value of the block's acceleration magnitude  $a$ ?

**Calculating  $F_N$ :** Because we need the magnitude  $f_k$  of the frictional force, we first must calculate the magnitude  $F_N$  of the normal force. Figure 6-4b is a free-body diagram showing the forces along the vertical  $y$  axis. The normal force is upward, the gravitational force  $\vec{F}_g$  with magnitude  $mg$  is downward, and (note) the vertical component  $F_y$  of the applied force is upward. That component is shown in Fig. 6-4c, where we can see that  $F_y = F \sin \theta$ . We can write Newton's second law ( $\vec{F}_{\text{net}} = m\vec{a}$ ) for those forces along the  $y$  axis as

$$F_N + F \sin \theta - mg = m(0),$$

$$F_N = mg - F \sin \theta.$$

**Calculating acceleration  $a$ :** Figure 6-4d is a free-body diagram for motion along the  $x$  axis. The horizontal component  $F_x$  of the applied force is rightward; from Fig. 6-4c, we see that  $F_x = F \cos \theta$ . The frictional force has magnitude  $f_k$  ( $= \mu_k F_N$ ) and is leftward. Writing Newton's second law for motion along the  $x$  axis gives us



$$F \cos \theta - \mu_k F_N = ma.$$

$$a = \frac{F}{m} \cos \theta - \mu_k \left( g - \frac{F}{m} \sin \theta \right).$$

**Finding a maximum:** To find the value of  $\theta$  that maximizes  $a$ , we take the derivative of  $a$  with respect to  $\theta$  and set the result equal to zero:

$$\frac{da}{d\theta} = -\frac{F}{m} \sin \theta + \mu_k \frac{F}{m} \cos \theta = 0.$$

$$\tan \theta = \mu_k.$$

$$\theta = \tan^{-1} \mu_k$$

$$= 21.8^\circ \approx 22^\circ.$$

## 6.4: The drag force and terminal speed

When there is a relative velocity between a fluid and a body (either because the body moves through the fluid or because the fluid moves past the body), the body experiences a **drag force,  $F_D$ , that opposes the relative motion and points in the direction in which the fluid flows relative to the body.**



**Fig. 6-5** This skier crouches in an “egg position” so as to minimize her effective cross-sectional area and thus minimize the air drag acting on her. (*Karl-Josef Hildenbrand/dpa/Landov LLC*)

## 6.4: Drag force and terminal speed

For cases in which air is the fluid, and the body is blunt (like a baseball) rather than slender (like a javelin), and the relative motion is fast enough so that the air becomes turbulent (breaks up into swirls) behind the body,

$$D = \frac{1}{2}C\rho Av^2,$$

where  $\rho$  is the air density (mass per volume),  $A$  is the **effective cross-sectional area of the body (the area of a cross section taken perpendicular to the velocity)**, and  $C$  is the drag coefficient .

When a blunt body falls from rest through air, the drag force is directed upward; its magnitude gradually increases from zero as the speed of the body increases. From Newton's second law along  $y$  axis

$$D - F_g = ma,$$

where  $m$  is the mass of the body. Eventually,  $a = 0$ , and the body then falls at a constant speed, called the **terminal speed  $v_t$**  .

$$\frac{1}{2}C\rho Av_t^2 - F_g = 0,$$

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

## 6.4: Drag force and terminal speed

# Some typical values of terminal speed

TABLE 6-1

### Some Terminal Speeds in Air

Object	Terminal Speed (m/s)	95% Distance <sup>a</sup> (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3

<sup>a</sup>This is the distance through which the body must fall from rest to reach 95% of its terminal speed.

Source: Adapted from Peter J. Brancazio, *Sport Science*, 1984, Simon & Schuster, New York.

## Example: terminal speed

A raindrop with radius  $R = 1.5$  mm falls from a cloud that is at height  $h = 1200$  m above the ground. The drag coefficient  $C$  for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water  $\rho_w$  is  $1000$  kg/m<sup>3</sup>, and the density of air  $\rho_a$  is  $1.2$  kg/m<sup>3</sup>.

(a) As Table 6-1 indicates, the raindrop reaches terminal speed after falling just a few meters. What is the terminal speed?

### KEY IDEA

The drop reaches a terminal speed  $v_t$  when the gravitational force on it is balanced by the air drag force on it, so its acceleration is zero. We could then apply Newton's second law and the drag force equation to find  $v_t$ , but Eq. 6-16 does all that for us.

**Calculations:** To use Eq. 6-16, we need the drop's effective cross-sectional area  $A$  and the magnitude  $F_g$  of the gravitational force. Because the drop is spherical,  $A$  is the area of a circle ( $\pi R^2$ ) that has the same radius as the sphere. To find  $F_g$ , we use three facts: (1)  $F_g = mg$ , where  $m$  is the drop's mass; (2) the (spherical) drop's volume is  $V = \frac{4}{3}\pi R^3$ ; and (3) the density of the water in the drop is the mass per volume, or  $\rho_w = m/V$ . Thus, we find

$$F_g = V\rho_w g = \frac{4}{3}\pi R^3 \rho_w g.$$

We next substitute this, the expression for  $A$ , and the given data into Eq. 6-16. Being careful to distinguish between the air den-

sity  $\rho_a$  and the water density  $\rho_w$ , we obtain

$$\begin{aligned} v_t &= \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}} \\ &= \sqrt{\frac{(8)(1.5 \times 10^{-3} \text{ m})(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}{(3)(0.60)(1.2 \text{ kg/m}^3)}} \\ &= 7.4 \text{ m/s} \approx 27 \text{ km/h.} \end{aligned} \quad (\text{Answer})$$

Note that the height of the cloud does not enter into the calculation.

(b) What would be the drop's speed just before impact if there were no drag force?

### KEY IDEA

With no drag force to reduce the drop's speed during the fall, the drop would fall with the constant free-fall acceleration  $g$ , so the constant-acceleration equations of Table 2-1 apply.

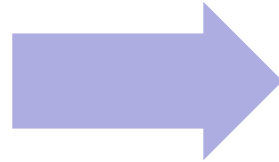
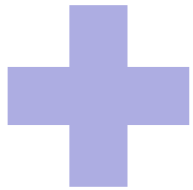
**Calculation:** Because we know the acceleration is  $g$ , the initial velocity  $v_0$  is 0, and the displacement  $x - x_0$  is  $-h$ , we use Eq. 2-16 to find  $v$ :

$$\begin{aligned} v &= \sqrt{2gh} = \sqrt{(2)(9.8 \text{ m/s}^2)(1200 \text{ m})} \\ &= 153 \text{ m/s} \approx 550 \text{ km/h.} \end{aligned} \quad (\text{Answer})$$

Had he known this, Shakespeare would scarcely have written, "it droppeth as the gentle rain from heaven, upon the place beneath." In fact, the speed is close to that of a bullet from a large-caliber handgun!

## 6.5: Uniform circular motion

The speed of  
the particle is  
constant



A particle  
travels around  
a circle/circular  
arc

Uniform  
circular  
motion

## 6.5: Uniform circular motion

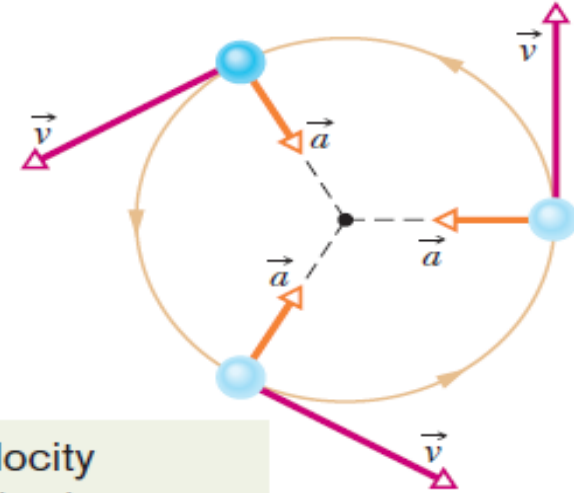
A body (represented as a particle) moving with speed  $v$  in uniform circular motion experiences an acceleration directed towards the center of the circle of radius  $r$ . The direction of the velocity of the particle changes, thus there is an acceleration (change in velocity). This center-seeking acceleration is called a centripetal acceleration.

$$a = \frac{v^2}{r} \quad (\text{centripetal acceleration}),$$

### Examples:

1. When a car moves in the circular arc, it has an acceleration that is directed toward the center of the circle. The frictional force due to the interaction of the tires and road provide the net of centripetal force responsible for this.
2. In a space shuttle orbiting the earth, both the rider and the shuttle are in uniform circular motion and experience an acceleration directed toward the center of the circular orbit. The net force, also called centripetal force, that causes this acceleration, is supplied by the gravitational pull exerted by Earth and directed radially inward, toward the center of Earth.

The acceleration vector always points toward the center.



The velocity vector is always tangent to the path.

# 6.5: Uniform circular motion

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = (-v \sin \theta) \hat{i} + (v \cos \theta) \hat{j}.$$

→ 
$$\vec{v} = \left( -\frac{vy_p}{r} \right) \hat{i} + \left( \frac{vx_p}{r} \right) \hat{j}.$$

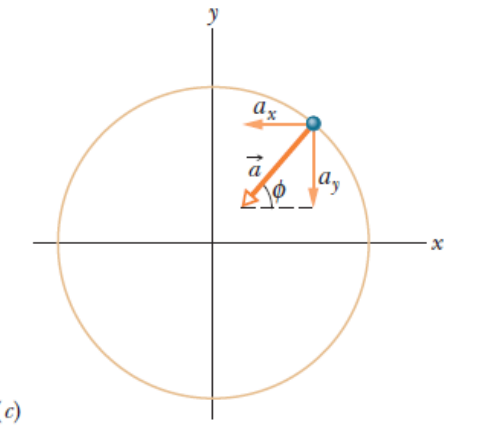
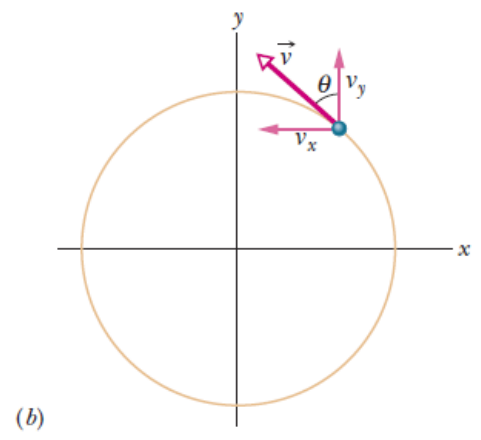
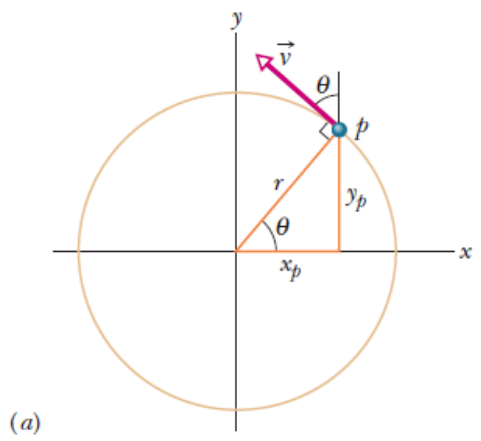
$$\vec{a} = \frac{d\vec{v}}{dt} = \left( -\frac{v}{r} \frac{dy_p}{dt} \right) \hat{i} + \left( \frac{v}{r} \frac{dx_p}{dt} \right) \hat{j}.$$

Note:  $\frac{dy_p}{dt} = v_y, \quad \frac{dx_p}{dt} = v_x$   
 $v_x = -v \sin \theta, \quad v_y = -v \cos \theta$

→ 
$$\vec{a} = \left( -\frac{v^2}{r} \cos \theta \right) \hat{i} + \left( -\frac{v^2}{r} \sin \theta \right) \hat{j}.$$

→ 
$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r} \sqrt{(\cos \theta)^2 + (\sin \theta)^2} = \frac{v^2}{r} \sqrt{1} = \frac{v^2}{r},$$

→ 
$$\tan \phi = \frac{a_y}{a_x} = \frac{-(v^2/r) \sin \theta}{-(v^2/r) \cos \theta} = \tan \theta.$$






# Example: top gun pilots

“Top gun” pilots have long worried about taking a turn too tightly. As a pilot’s body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is  $2g$  or  $3g$ , the pilot feels heavy. At about  $4g$ , the pilot’s vision switches to black and white and narrows to “tunnel vision.” If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as  $g$ -LOC for “ $g$ -induced loss of consciousness.”

What is the magnitude of the acceleration, in  $g$  units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of  $\vec{v}_i = (400\hat{i} + 500\hat{j})$  m/s and 24.0 s later leaves the turn with a velocity of  $\vec{v}_f = (-400\hat{i} - 500\hat{j})$  m/s? 

## KEY IDEAS

We assume the turn is made with uniform circular motion. Then the pilot’s acceleration is centripetal and has magnitude  $a$  given by

$$a = v^2/R.$$

Also, the time required to complete a full circle is the period given by  $T = 2\pi R/v$

## Calculations:

Because we do not know radius  $R$ , let’s solve for  $R$  from the period equation for  $R$  and substitute into the acceleration eqn.

$$a = \frac{2\pi v}{T}.$$

Speed  $v$  here is the (constant) magnitude of the velocity during the turning.

$$v = \sqrt{(400 \text{ m/s})^2 + (500 \text{ m/s})^2} = 640.31 \text{ m/s}.$$

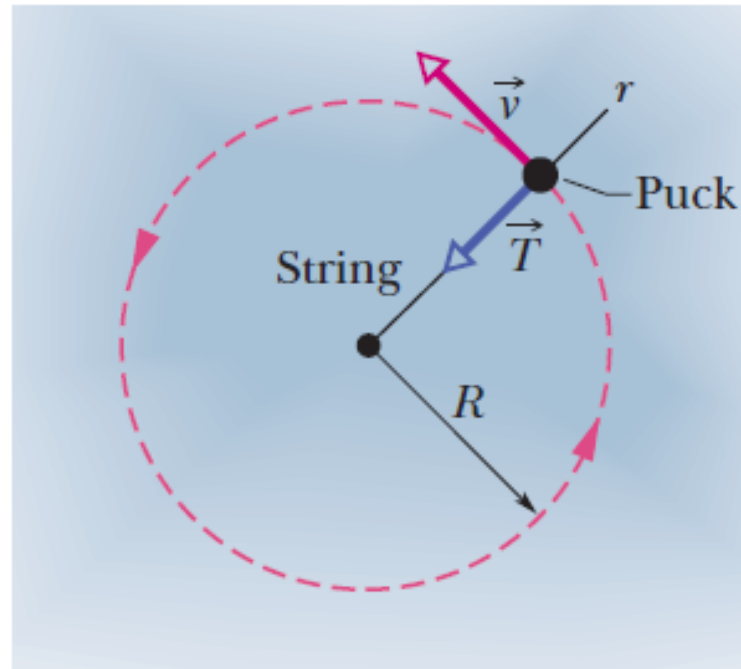
To find the period  $T$  of the motion, first note that the final velocity is the reverse of the initial velocity. This means the aircraft leaves on the opposite side of the circle from the initial point and must have completed half a circle in the given 24.0 s.

Thus a full circle would have taken  $T = 48.0$  s. Substituting these values into our equation for  $a$ :

$$a = \frac{2\pi(640.31 \text{ m/s})}{48.0 \text{ s}} = 83.81 \text{ m/s}^2 \approx 8.6g. \quad (\text{Answer})$$

## 6.5: Uniform circular motion

Example: hockey puck:



**Fig. 6-8**

An overhead view of a hockey puck moving with constant speed in a circular path of radius  $R$  on a horizontal frictionless surface. The centripetal force on the puck is  $T$ , the pull from the string, directed inward along the radial axis  $r$  extending through the puck.

## 6.5: Uniform circular motion

Centripetal force: Net or resultant force acting on a body (particle). Centripetal force is the resultant of applied forces such as tension (string, rope, cable, cord), friction (tires interacting with road), and gravity (one body orbiting another). The effect of a known force which supplies the centripetal force is a centripetal acceleration; a particle accelerates as the direction of the velocity changes without changing the particle's speed. The direction of the acceleration points toward the center of the circle that describes the motion.

### Newton's 2<sup>nd</sup> Law:

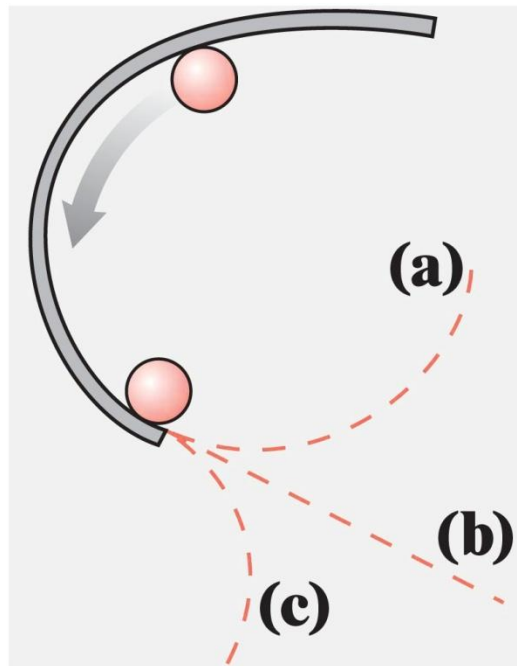
$$F = m \frac{v^2}{R} \quad (\text{magnitude of centripetal force}).$$

*Since the speed  $v$  here is constant, the magnitudes of the acceleration and the force are also constant.*



# Clicker question

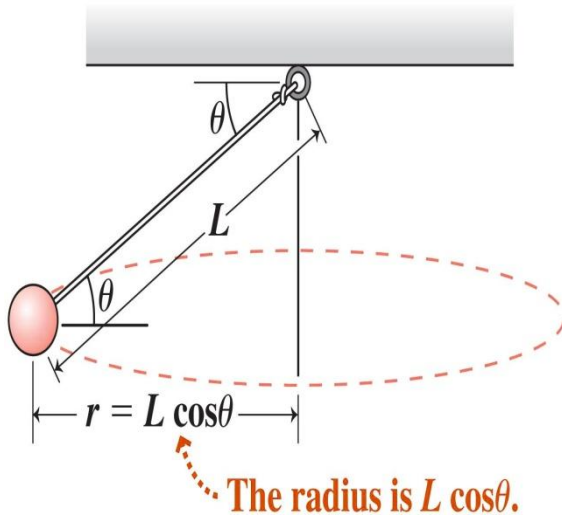
- On a horizontal tabletop is a curved barrier that exerts a force on a ball, guiding its motion in a circular path as shown. After the ball leaves the barrier, which of the dashed paths shown does it follow?



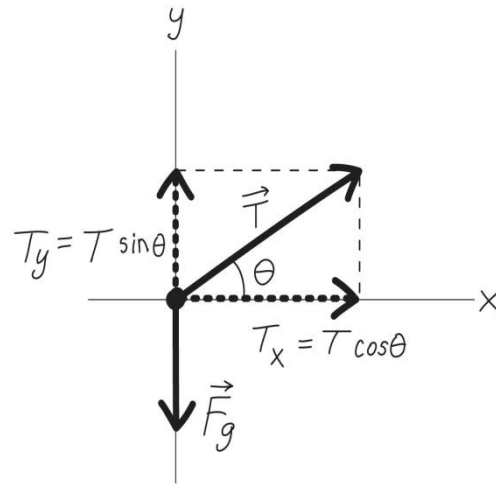
## 6.5: Uniform circular motion

- Problems involving circular motion are no different from other Newton's law problems.
- Identify the forces acting on an object.
- Draw a free body diagram.
- Write Newton's 2<sup>nd</sup> law.
- In this case, the acceleration has magnitude  $v^2/r$  and the direction points toward the center of the circle.

A ball whirling on a string:



Free body diagram:



Newton's 2<sup>nd</sup> law:

$$\vec{T} + \vec{F}_g = m\vec{a}$$

*In components:*

$$F_x : T \cos \theta = \frac{mv^2}{L \cos \theta}$$

$$F_y : T \sin \theta - mg = 0$$

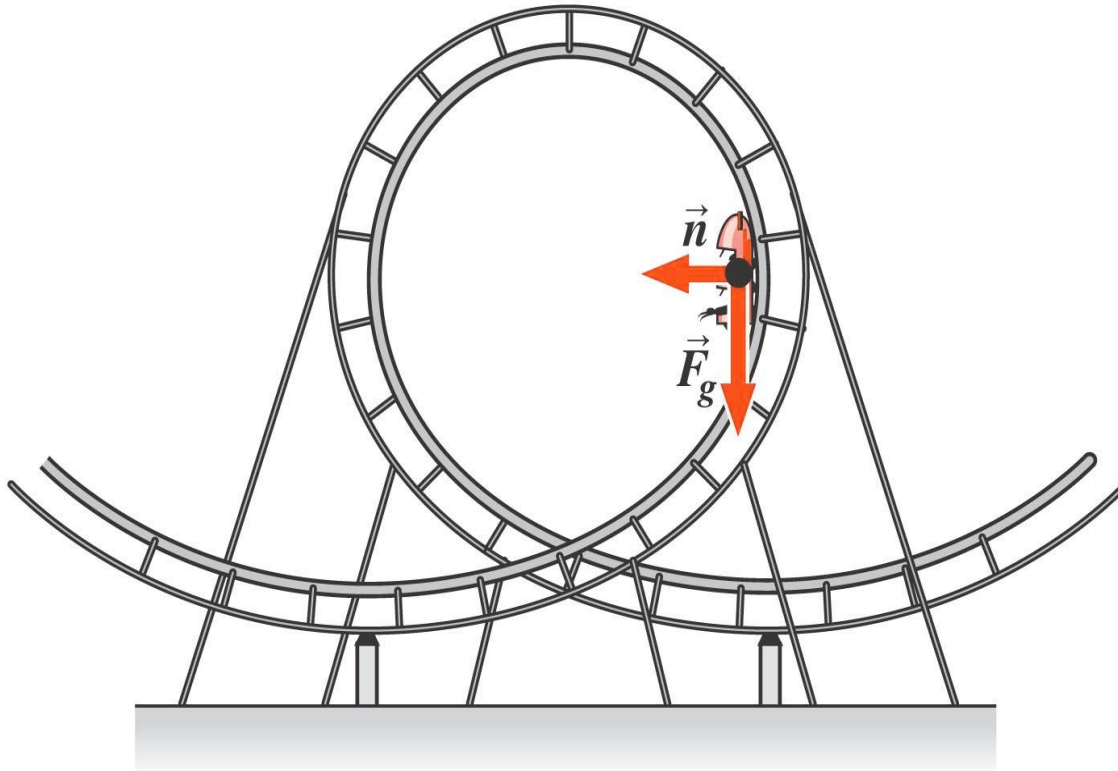
Solve for the ball's speed:

$$v = \sqrt{\frac{TL \cos^2 \theta}{m}} = \sqrt{\frac{(mg / \sin \theta)L \cos^2 \theta}{m}} = \sqrt{\frac{gL \cos^2 \theta}{\sin \theta}}$$

## 6.5: Uniform circular motion

# Loop-the-Loop!

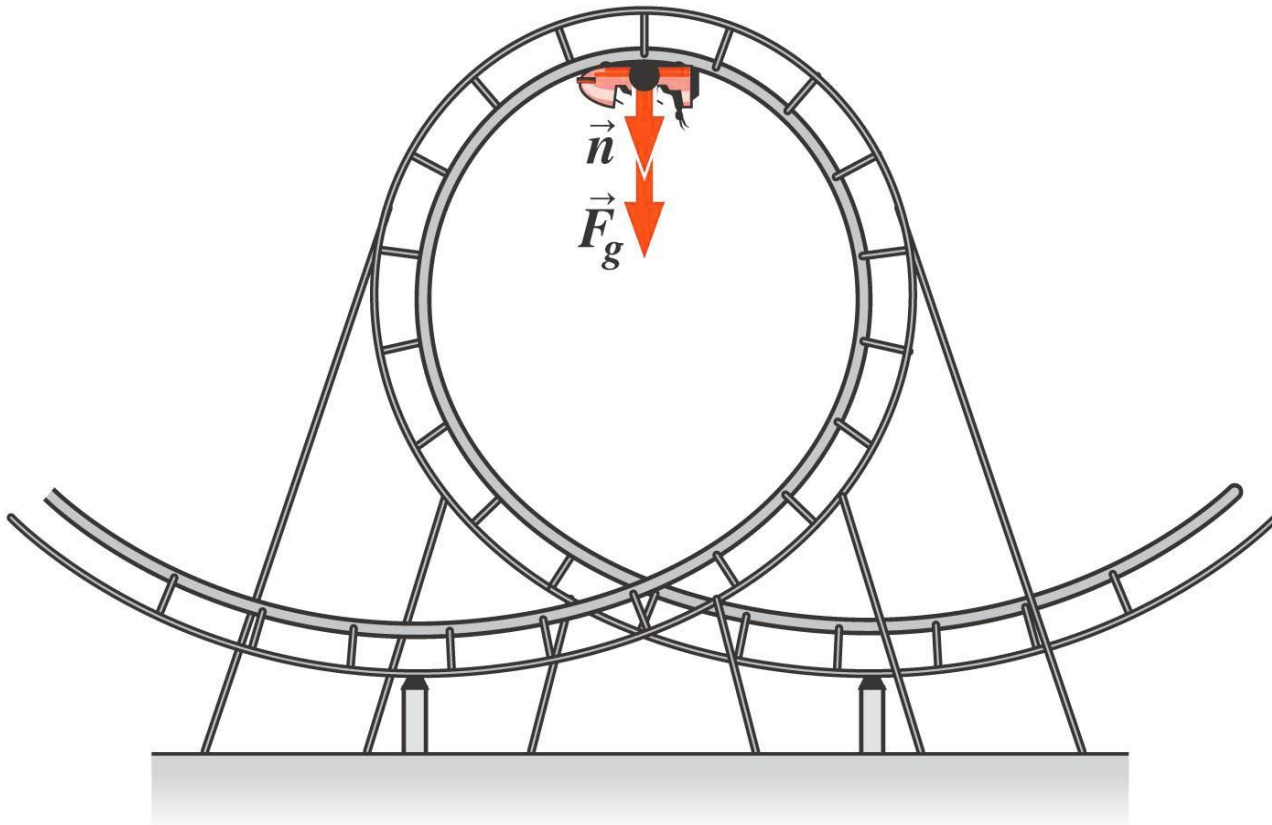
- The two forces acting on the car are gravity and the normal force.
- Gravity is always downward, and the normal force is perpendicular to the track.
- Here the two are at right angles:
  - The normal force acts perpendicular to the car's path, such that the direction of motion continually changes.
  - Gravity acts opposite the car's velocity, slowing the car.



## 6.5: Uniform circular motion

# Loop-the-Loop!

- Now both forces are downward:
  - For the car to stay in contact with the track, the normal force must be greater than zero.
  - The minimum speed is the speed that allows the normal force to get arbitrarily close to zero at the top of the loop.
  - Then gravity alone provides the force that maintains the car in circular motion.



## 6.5: Uniform circular motion

# Loop-the-Loop!

- Therefore Newton's 2<sup>nd</sup> law has a single component, with the gravitational force  $mg$  providing the acceleration  $v^2/r$  that holds the car in its circular path:

$$\vec{F} = m\vec{a} \quad \rightarrow \quad mg = \frac{mv^2}{r}$$

- Solving for the minimum (critical) speed at the loop top yields:  $v = \sqrt{gr}$
- Slower than this critical speed at the top, and the car will leave the track.
- Since this result is independent of mass, car and passengers will all remain on the track as long as:

$$v \geq \sqrt{gr}$$





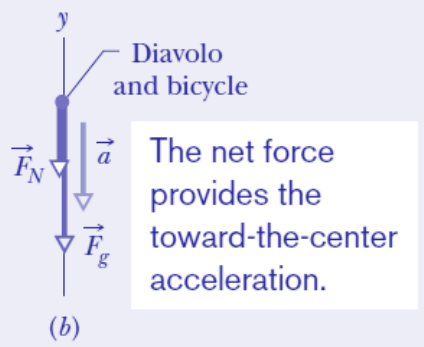
# Example: Vertical circular loop

In a 1901 circus performance, Allo “Dare Devil” Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. 6-9a). Assuming that the loop is a circle with radius  $R = 2.7$  m, what is the least speed  $v$  that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there?



(a)

Fig. 6-9



(b)

## KEY IDEA

We can assume that Diavolo and his bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration  $\vec{a}$  of this particle must have the magnitude  $a = v^2/R$  given by Eq. 6-17 and be directed downward, toward the center of the circular loop.

**Calculations:** The forces on the particle when it is at the top of the loop are shown in the free-body diagram of Fig 6-9b. The gravitational force  $\vec{F}_g$  is downward along a  $y$  axis; so is the normal force  $\vec{F}_N$  on the particle from the loop; so also is the centripetal acceleration of the particle. Thus, Newton’s second law for  $y$  components ( $F_{net,y} = ma_y$ ) gives us

$$-F_N - F_g = m(-a)$$

and

$$-F_N - mg = m\left(-\frac{v^2}{R}\right).$$

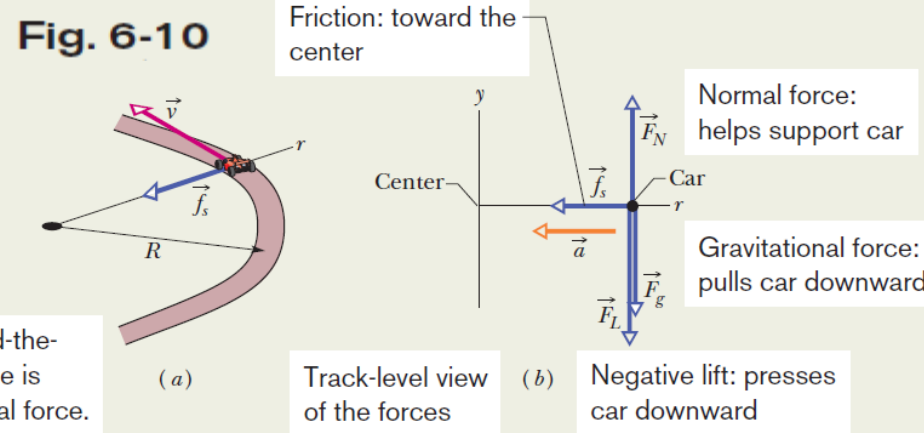
If the particle has the *least speed*  $v$  needed to remain in contact, then it is on the *verge of losing contact* with the loop (falling away from the loop), which means that  $F_N = 0$  at the top of the loop (the particle and loop touch but without any normal force). Substituting 0 for  $F_N$  in Eq. 6-19, solving for  $v$ , and then substituting known values give us

$$v = \sqrt{gR} = \sqrt{(9.8 \text{ m/s}^2)(2.7 \text{ m})} = 5.1 \text{ m/s.} \quad (\text{Answer})$$

# Example: car in flat circular turn

Figure 6-10a represents a Grand Prix race car of mass  $m = 600 \text{ kg}$  as it travels on a flat track in a circular arc of radius  $R = 100 \text{ m}$ . Because of the shape of the car and the wings on it, the passing air exerts a negative lift  $\vec{F}_L$  downward on the car. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.)

The toward-the-center force is the frictional force.



**Radial calculations:** The frictional force  $\vec{f}_s$  is shown in the free-body diagram of Fig. 6-10b. It is in the negative direction of a radial axis  $r$  that always extends from the center of curvature through the car as the car moves. The force produces a centripetal acceleration of magnitude  $v^2/R$ . We can relate the force and acceleration by writing Newton's second law for components along the  $r$  axis ( $F_{\text{net},r} = ma_r$ ) as

$$-f_s = m \left( -\frac{v^2}{R} \right).$$

Substituting  $f_{s,\text{max}} = \mu_s F_N$  for  $f_s$  leads us to

$$\mu_s F_N = m \left( \frac{v^2}{R} \right).$$

## Combining results:

**Vertical calculations:** Next, let's consider the vertical forces on the car. The normal force  $\vec{F}_N$  is directed up, in the positive direction of the  $y$  axis in Fig. 6-10b. The gravitational force  $\vec{F}_g = m\vec{g}$  and the negative lift  $\vec{F}_L$  are directed down. The acceleration of the car along the  $y$  axis is zero. Thus we can write Newton's second law for components along the  $y$  axis ( $F_{\text{net},y} = ma_y$ ) as

$$F_N - mg - F_L = 0,$$

$$F_N = mg + F_L.$$

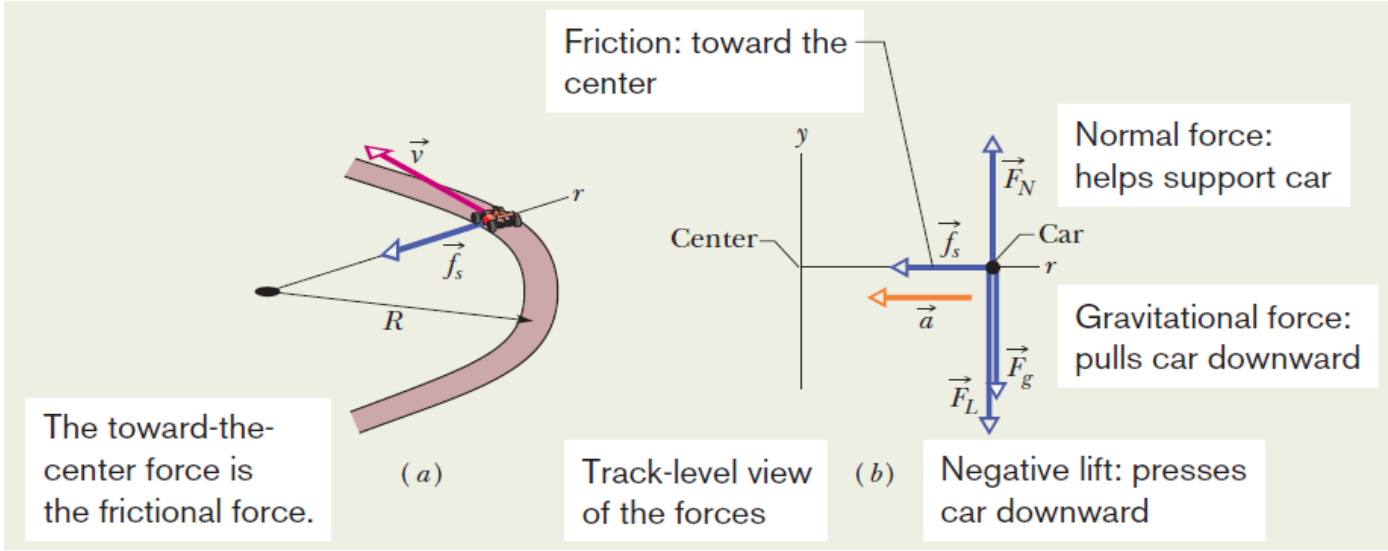
or

$$F_L = m \left( \frac{v^2}{\mu_s R} - g \right)$$

$$= (600 \text{ kg}) \left( \frac{(28.6 \text{ m/s})^2}{(0.75)(100 \text{ m})} - 9.8 \text{ m/s}^2 \right)$$

$$= 663.7 \text{ N} \approx 660 \text{ N.} \quad \text{(Answer)}$$

**Example: car in flat circular turn, cont.**



(b) The magnitude  $F_L$  of the negative lift on a car depends on the square of the car's speed  $v^2$ , just as the drag force does. Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?

**Calculations:** Thus we can write a ratio of the negative lift  $F_{L,90}$  at  $v = 90$  m/s to our result for the negative lift  $F_L$  at  $v = 28.6$  m/s as

$$\frac{F_{L,90}}{F_L} = \frac{(90 \text{ m/s})^2}{(28.6 \text{ m/s})^2}$$

Using  $F_L = 663.7 \text{ N}$ ,

$$F_{L,90} = 6572 \text{ N} \approx 6600 \text{ N}.$$

**Upside-down racing:** The gravitational force is, of course, the force to beat if there is a chance of racing upside down:

$$F_g = mg = (600 \text{ kg})(9.8 \text{ m/s}^2) = 5880 \text{ N}.$$

# Summary

- The problem solving approach to all Newton's law problems is the same.
- Strategy:
  - Identify all the forces acting on the object or objects of interest.
  - Draw a free body diagram.
  - Establish a coordinate system.
  - Write Newton's law in vector form by equating the net force to the product of mass and acceleration.
  - Write Newton's law in components.
  - Solve for the quantities of interest.